1. For the group $GL(2, \mathbb{Z}_3)$, which one of the following is ***false***?

(A) $GL(2, \mathbb{Z}_3)$ is a non-Abelian group. (B) $o(GL(2, \mathbb{Z}_3)) = 81$.

(C) $o\left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\right) = 3$. (D) $\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.

2. If H and K are two subgroups of a group G, then which one of the following statements is, in general, not ***true***? ($HK = \{hk : h \in H, k \in K\}$)

(A) If $H \subseteq K$ or $K \subseteq H$, then HK is a subgroup of G

(B) If $HK = KH$, then HK is a subgroup of G

(C) If either H or K is a normal subgroup G, then HK is a normal subgroup of G.

(D) If G is a commutative group, then HK is a normal subgroup of G.

3. For a group G, consider the following statements.

I. If $\dfrac{G}{Z(G)}$ is a commutative group, then G is a commutative group.

II. If $\dfrac{G}{Z(G)}$ is a cyclic group, then G is a commutative group.

Pick out the ***correct*** answer from the following options.

(A) I is false, but II is true. (B) I is true, but II is false.

(C) Both I and II are false. (D) Both I and II are true.

4. Which on of the following statements is ***false***?

(A) The quotient group $\dfrac{S_n}{A_n}$ is isomorphic to $\mathbb{Z}_2$.

(B) A_n is a non-commutative group for $n \geq 4$.

(C) $Z(S_n) = \{e\}$ for every $n \geq 3$, where e is the identity element in S_n.

(D) A_n is a simple group for every $n \geq 3$.

5. Let G be a group and $a \in G$. If $a^2 \neq e$ and $a^6 = e$, then which one of the following must be ***true***? (e is the identity element of G)

(A) $a^3 = e$ and $a^4 \neq e$. (B) $a^4 \neq e$ and $a^5 \neq e$.

(C) $a^4 \neq e$ and $a^5 = e$. (D) $a^3 \neq e$ and $a^4 = e$.

6. How many Sylow 5-subgroup(s) is/are there in a group of order 40?

(A) 5 (B) 4

(C) 2 (D) 1

7. Which one of the following statements is *false*? (Char(R) denotes the characteristic of a ring R)

(A) $\mathbf{Z}$ is a subring of $\mathbf{Q}$. (B) Char $(M_2(\mathbf{Z})) = 0$

(C) $\mathbf{Z}$ is an ideal of $\mathbf{Q}$. (D) $\mathbf{Z}[\sqrt{-5}]$ is an integral domain.

8. For what value of $\lambda \in \mathbf{Z}_3$, the quotient ring $\dfrac{\mathbf{Z}_3[x]}{<x^3+x^2+\lambda>}$ is a field?

(A) 3 (B) 2

(C) 1 (D) 0

9. The field $\mathbf{Q}(\sqrt{2}+\sqrt{3})$ is an algebraic extension of the field $\mathbf{Q}$ of degree ________.

(A) 2 (B) 4

(C) 6 (D) 8

10. For the polynomials $p(x) = x^3 + 2x^2 + 1$ and $q(x) = 2x^2 + x + 2$, which one of the following statements is *true*?

(A) $p(x)$ is irreducible over $\mathbf{Z}_3$, but not $q(x)$.

(B) $q(x)$ is irreducible over $\mathbf{Z}_3$, but not $p(x)$.

(C) Both $p(x)$ and $q(x)$ are irreducible over $\mathbf{Z}_3$.

(D) Neither $p(x)$ nor $q(x)$ are irreducible over $\mathbf{Z}_3$.

11. $\dfrac{30!}{7}$ (mod 31) $\equiv$ ________.

(A) 27 (mod 31) (B) 25 (mod 31)

(C) 24 (mod 31) (D) 22 (mod 31)

12. If p is a prime number and $d \mid (p-1)$, then the congruence: $x^d \equiv 1 \pmod{p}$ has ________.

$x^6 \equiv 1 \pmod{13}$

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(A) no solution.

(B) at most d incongruent solution(s).

(C) exactly d incongruent solution(s).

(D) at least $(p-d-1)$ incongruent solution(s).

13. On the ring $R = \left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix} : a, b \in \mathbf{Z}\right\}$ under matrix addition and matrix multiplication, define $\phi : R \to \mathbf{Z}$ by $\phi\left(\begin{bmatrix} a & b \\ b & a \end{bmatrix}\right) = a - b$. Then, which one of the following statements is *false*?

(A) ϕ is a ring homomorphism. (B) $\dfrac{R}{\mathrm{Ker}(\phi)} \cong \mathbf{Z}$.

(C) $\mathrm{Ker}(\phi)$ is a prime ideal of R. (D) $\mathrm{Ker}(\phi)$ is a maximal ideal of R.

Contd.

14. Consider the following classes of commutative rings with unity.

ED is the class of Euclidean domain, PID is the class of Principal ideal domain, UFD is the class of Unique factorization domain and ID is the class of Integral domain.

Choose the ***correct*** containment relations from the options given below.

(A) PID $\subset$ ED $\subset$ UFD $\subset$ ID.

(B) ED $\subset$ UFD $\subset$ PID $\subset$ ID.

(C) ED $\subset$ PID $\subset$ UFD $\subset$ ID.

(D) UFD $\subset$ PID $\subset$ ED $\subset$ ID.

15. Let $S = \{v_1, v_2, v_3\}$ be a basis of a vector space V over the field $\mathbb{R}$. Consider the following subsets of V.

$S_1 = \{2v_1+3v_2,\ 2v_1-v_3,\ v_1+v_2\}$, $S_3 = \{v_1+2v_2-2v_3,\ v_1+v_2+v_3,\ 3v_1+4v_2\}$,

$S_2 = \{2v_1+3v_2,\ 3v_1-v_3,\ v_1-3v_2-v_3\}$, $S_4 = \{6v_1-3v_2+v_3,\ 3v_1+4v_2+v_3,\ v_1+v_3\}$.

Choose the ***correct*** answer from the options given below.

(A) Only S_1 and S_4 are bases of V.

(B) Only S_1, S_2 and S_4 are bases of V.

(C) Only S_2 and S_3 are bases of V.

(D) Only S_1, S_3 and S_4 are bases of V.

16. If

$$W_1 = \{(x_1, x_2, \ldots, x_{10}) \in \mathbb{R}^{10} : x_k = 0 \text{ when } 2 \text{ divides } k,\ 1 \le k \le 10\} \text{ and}$$

$$W_2 = \{(x_1, x_2, \ldots, x_{10}) \in \mathbb{R}^{10} : x_k = 0 \text{ when } 5 \text{ divides } k,\ 1 \le k \le 10\}$$

are subspaces of the real vector space $\mathbb{R}^{10}$, then $\dim(W_1 \cap W_2) =$ ________.

(A) 1

(B) 4

(C) 5

(D) 7

17. If T is the linear transformation on the vector space $\mathbb{R}^2$ over the field $\mathbb{R}$ that reflects the points of $\mathbb{R}^2$ across the line $y = -x$, then which one of the following matrix represents T with respect to the basis $\{(1,0),(0,1)\}$ of $\mathbb{R}^2$?

(A) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

18. For the linear transformation $T : P_2(\mathbb{R}) \to \mathbb{R}^2$ defined by

$$T(p(x)) = (p(1), p(-1)) \text{ for all } p \in P_2(\mathbb{R}),$$

which one of the following statements is *not* necessarily *true*?

(A) Range of T is $\mathbb{R}^2$.

(B) T is one-to-one.

(C) Kernel of T is span$\{x^2 - 1\}$.

(D) T is onto.

19. Which one of the following statements is a necessary and sufficient condition for two matrices in $M_3(\mathbb{R})$ to be similar?

(A) They have the same characteristic polynomial.

(B) They have the same minimal polynomial.

(C) They have the same determinant and trace.

(D) They have the same minimal amd characteristic polynomial.

20. If $M \in M_3(\mathbb{R})$ and $\det(M) = 0$, then which one of the following statements is *true*? ($\det(M)$ denotes the determinant of the matrix M)

(A) 0 is not an eigenvalue of M^2.

(B) 0 is an eigenvalue of M, but 0 is not an eigenvalue of M^2.

(C) Rank of M^3 is strictly less than n.

(D) M is a nilpotent matrix.

21. Which one of the following sets is countable?

(A) $S_1 = \{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Q} \text{ or } x + y \in \mathbb{Q}\}$.

(B) $S_2 = \{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Q} \text{ and } y \in \mathbb{Q}\}$.

(C) $S_3 = \{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Q} \text{ or } y \in \mathbb{Q}\}$.

(D) $S_4 = \{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Q} \text{ or } y^2 \in \mathbb{Q}\}$.

Contd.

22. If d_1 and d_2 are metrics on a non-empty set X, then which one of the following statements is *true*?

(A) $d_1 \cdot d_2$ is a metric on X.

(B) $d_1 + \lambda d_2$ ($\lambda \in \mathbb{R}$) is a metric on X.

(C) min $\{d_1, d_2\}$ is a metric on X.

(D) max $\{d_1, d_2\}$ is a metric on X.

23. On $\mathbb{R}$, consider the following sets

$T_1 = \{\phi\} \cup \{\mathbb{R}\} \cup \{(-\infty, a] : a \in \mathbb{R}\}$ and $T_2 = \{\phi\} \cup \{\mathbb{R}\} \cup \{(-\infty, a) : a \in \mathbb{R}\}$.

Choose the *correct* answer from the options given below.

(A) T_1 is a topology on $\mathbb{R}$.

(B) T_2 is a topology on $\mathbb{R}$

(C) Both T_1 and T_2 are topologies on $\mathbb{R}$.

(D) Neither T_1 nor T_2 is a topology on $\mathbb{R}$.

24. Let $X = \{a, b, c\}$ and let $T = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ be a topology on X. Then, the set of all limit point(s) of the set $S = \{a, b\}$ is ________.

(A) X

(B) $\{a, c\}$

(C) $\{c\}$

(D) $\{b, c\}$

25. The set $S = \{(x,y) \in \mathbb{R}^2 : xy < 0\}$ is ________.

(A) connected, but not a compact subset of $\mathbb{R}^2$.

(B) compact, but not a connected subset of $\mathbb{R}^2$.

(C) both connected and compact subset of $\mathbb{R}^2$.

(D) neither connected nor a compact subset of $\mathbb{R}^2$.

26. Let $\{x_n\}_{n \geq 1}$ be a sequence of positive numbers such that $\lim_{n \to \infty} x_n = x$. Then, the sequence $\{y_n\}_{n \geq 1}$ defined by

$$y_n = \frac{1}{n}\left\{x_1\left(1+\frac{x}{n}\right)^n + x_2\left(1+\frac{x}{n-1}\right)^{n-1} + \dots + x_n(1+x)\right\}$$

(A) converges to e^x.

(B) converges to ex.

(C) converges to xe^x.

(D) does not converge.

27. Which one of the following conditions imply that the sequence $\{x_n\}_{n\geq 1}$ of real numbers is a Cauchy sequence?

(A) $|x_{n+1} - x_n| \to 0$ as $n \to \infty$.

(B) $|x_{n+1} - x_n| < |x_n - x_{n-1}|$ for each $n \geq 2$.

(C) $|x_{n+1} - x_n| \leq \frac{1}{n}$ for each $n \geq 1$.

(D) $|x_{n+1} - x_n| \leq \frac{1}{n^2}$ for each $n \geq 1$.

28. Which one of the following statements is ***true*** for the function f defined on $\mathbb{R}$ by $f(x) = |x - \pi| (e^{|x|} - 1) \sin|x|$?

(A) f is differentiable at all points of $\mathbb{R}$.

(B) f is differentiable at all points of $\mathbb{R}$ except $x = 0$.

(C) f is differentiable at all points of $\mathbb{R}$ except $x = \pi$.

(D) f is differentiable at all points of $\mathbb{R}$ except $x = 0$ and $x = \pi$.

29. Consider the following statements.

I. The series : $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\sqrt{n}}$ is absolutely convergent.

II. The series : $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ is conditionally convergent.

Choose the ***correct*** answer from the following options.

(A) Only I is true.

(B) Only II is true.

(C) Both I and II are true.

(D) Neither I nor II is true.

30. $\int_0^{10} x\,d(x+[x]) =$ ______ ($[x]$ denotes the greatest integer $\leq x$).

(A) 45

(B) 50

(C) 100

(D) 105

Contd.

31. Consider the following statements.

I. The sequence $\{f_n\}_{n\geq 1}$ of functions defined by $f_n(x) = \dfrac{nx}{1+n^2x^2}$ converges uniformly on [0, 1].

II. The sequence $\{f_n\}_{n\geq 1}$ of functions defined by $f_n(x) = \dfrac{x}{1+nx^2}$ converges uniformly on [0, 1].

Pick out the ***correct*** answer from the options given below.

(A) Both I and II are true.
(B) I is true, but II is false.
(C) Neither I nor II is true.
(D) I is false, but II is true.

32. For the function $f(x, y) = x^3 + y^3 - 3xy$, which one of the following statements is ***true***?

(A) f has more than two critical points.
(B) (0, 0) is not a saddle point of f.
(C) f attains its minimum value at (1, 1).
(D) f attains its maximum value at (1, −1).

33. What is the value of the integral : $\int_C (xy\,dy - y^2 dx)$, where C is a square, cut from the first quadrant by the lines $x = 1$ and $y = 1$?

(A) $\dfrac{5}{3}$
(B) $\dfrac{3}{2}$
(C) 1
(D) $\dfrac{1}{2}$

34. The value of the integral : $\int_S \vec{F} \cdot d\vec{S}$ of the vector field $\vec{F} = (4x, 5y, 6z)$ over the entire surface of the cube S of unit edge length and sides parallel to the co-ordinate axes with its centroid at (1, 2, 3) is ________.

(A) 28
(B) 21
(C) 15
(D) 12

35. Which one of the following statements is not ***true***?

(A) A set with positive outer measure is always countable.
(B) On an uncountable set X, the function m^* defined on $P(X)$ (power set of X) by

$$m^*(E) = \begin{cases} 0, & \text{if } E \subseteq X \text{ is countable} \\ 1, & \text{if } E \subseteq X \text{ is uncountable} \end{cases}$$ is an outer measure on X.

(C) If the outer measure of $E \subset \mathbb{R}^2$ is zero, then the interior of E is an empty set.

(D) It the outer measure of the boundary of $E \subset \mathbb{R}^2$ is zero, then E is a measurable set.

36. Which one of the following statements is *false*?

(A) The Lebesgue measure of any straight line (finite as well as infinite) in $\mathbb{R}^2$ is zero.

(B) The Lebesgue measure of any curve in $\mathbb{R}^2$ is zero.

(C) The Lebesgue measure of any circle in $\mathbb{R}^2$ is its area.

(D) The Lebesgue measure of any open subset of $\mathbb{R}^3$ is its volume.

37. Consider the following statements.

I. If $f : \mathbb{R} \to \mathbb{R}$ is a Lebesgue measurable function and $g : \mathbb{R} \to \mathbb{R}$ is a continuous function, then $h = (g \circ f) : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = g(f(x))$ is a Lebesgue measurable function.

II. If $f : \mathbb{R} \to \mathbb{R}$ is a continuous function and $g : \mathbb{R} \to \mathbb{R}$ is a Lebesgue measurable function, then $h = (g \circ f) : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = g(f(x))$ is a Lebesgue measurable function.

Choose the ***correct*** answer from the following options.

(A) Only I is true.

(B) Only II is true.

(C) Both I and II are true.

(D) Neither I nor II is true.

38. Let (X, M, μ) be a measure space. If $\{f_n\}_{n\geq 1}$ is a sequence of non-negative measurable functions on X such that $\liminf_{n\to\infty} f_n(x) = f(x)$ for all $x \in X$, then which one of the following is *correct*?

(A) $\int_X f(x)d\mu = \liminf_{n\to\infty} \int_X f_n(x)d\mu$.

(B) $\int_X f(x)d\mu \neq \liminf_{n\to\infty} \int_X f_n(x)d\mu$.

(C) $\int_X f(x)d\mu \leq \liminf_{n\to\infty} \int_X f_n(x)d\mu$.

(D) $\int_X f(x)d\mu \geq \liminf_{n\to\infty} \int_X f_n(x)d\mu$.

39. Consider the function $f : [0, 1] \to \mathbb{R}$ defined by $f(x) = \begin{cases} 0, & x \in \mathbb{Q} \\ 1, & \text{otherwise} \end{cases}$. If $L(f)$, $R(f)$ denote the value of the Lebesgue integral of f and the value of the Riemann integral of f, then choose the ***correct*** answer from the following options.

(A) $L(f) = R(f) = 1$.

(B) $L(f) = R(f) = 0$.

(C) $L(f) = 1$ and $R(f) = 0$.

(D) $L(f) = 1$, but $R(f)$ does not exists.

40. Which of the following inclusion relations is/are ***true***?

I. $L^2([0, 1]) \subset L^1([0, 1])$.

II. $L^2([1, \infty]) \subset L^1([1, \infty])$.

(A) Both I and II.

(B) Neither I nor II.

(C) Only I.

(D) Only II.

$16 - 2 - 8 =$

41. A quadratic equation : $x^4 - x - 8 = 0$ is to be solved numerically by using Secant method. With the initial guesses $x_0 = 1$ and $x_1 = 2$, the approximate value of x_2 is ________.

(A) 1.571

(B) 2.358

(C) 2.538

(D) 2.853

$x_2 = \frac{1\times 6 + 8\times 2}{14} = \frac{22}{14} = 1.$

42. If the equation : $x^3 - 7x + 2 = 0$ has a solution in $[0, 1]$, then the rate of convergence of the sequence $\{x_n\}_{n\geq 1}$ defined by the iterative process :

$$x_{n+1} = \frac{1}{7}\left(x_n^3 + 2\right),\ n \geq 1$$

$x_{n+1} = \frac{x_n^3+2}{7} =$

is ________.

(A) 2

(B) $\frac{1+\sqrt{5}}{2}$

(C) $\frac{3}{2}$

(D) 1

43. If a, b and c are real numbers such that the following quadrature formula

$$\int_{-1}^{1} f(x)dx \approx a\, f(-1) + b\, f'(0) + c\, f'(1)$$

is exact for all polynomials of degree ≤ 2, then $a + b + c =$ ________.

(A) 7

(B) 5

(C) 4

(D) 3

44. Newton-Raphson method is used to find the root(s) of the equation: $x^2 - 2 = 0$. If the initial approximation $x_0 = -1$, then the sequence of iterations converges to

(A) -1 (B) $-\sqrt{2}$

(C) 1.4 (D) $\sqrt{2}$

45. Using the following data :

x	0	1	2	3
$f(x)$	1	0.5	0.2	0.1

$= \frac{h}{2}[1+0.1+2(0.5+0.2)]$

the value of the integral : $\int_0^3 f(x)dx$ by Trapezoidal rule is ________.

(A) 2 (B) 1.5

(C) 1.25 (D) 1.15

46. $\Delta^{10}\{(1-x)(1-x^2)(1-x^3)(1-x^4)\}$ = ________ (Δ denotes the forward difference operator).

(A) 10 (B) 10!

(C) $10!\,x$ (D) $9!\,x^{10}$

47. For a function f, let $(x_0, f(x_0)) = (0, -1)$, $(x_1, f(x_1)) = (1, a)$ and $(x_2, f(x_2)) = (2, b)$. If the first order divided differences $f[x_0, x_1] = 5$, $f[x_1, x_2] = c$ and the second order divided difference $f[x_0, x_1, x_2] = -\frac{3}{2}$, then the values of a, b, c are ________.

(A) 6, 2, 4 (B) 4, 2, 6

(C) 4, 6, 2 (D) 2, 4, 6

$\frac{f(x_1,x_2) - f(x_0,x_1)}{x_2 - x_0}$

$\frac{\frac{f(x_2)-f(x_1)}{x_2-x_1} - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}$

48. If the Lagrange polynomial p satisfying the following data :

x	15	18	22
$p(x)$	24	37	25

is given by $p(x) = 24\ell_0(x) + 37\ell_1(x) + 25\ell_2(x)$, then $\ell_1(16)$ = ________.

(A) 0.57 (B) 0.50

(C) 0.33 (D) 0.17

$\ell_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$

$= \frac{(16-15)(16-22)}{(18-15)(18-22)}$

$= \frac{-1\times 8}{-3\times 4}$

49. Match the ***correct*** pairs.

Numerical integration scheme	Order of fitting polynomial
P. Trapezoidal Rule	I. Third
Q. Simpson's $\frac{1}{3}$-rd Rule	II. First
R. Simpson's $\frac{3}{8}$-th Rule	III. Second

(A) P → I, Q → II, R → III
(B) P → II, Q → III, R → I
(C) P → III, Q → II, R → I
(D) P → II, Q → I, R → II

50. The approximate value of $y(0.1)$ by Euler's method for the differential equation:

$$\frac{dy}{dx} = x^2y - 1,\ y(0) = 1 \text{ is } \_\_\_\_\_\_.$$

$y = y_0 + hf(x_0, y_0)$

$= 1 - 0.1 \times$

(A) 0.75
(B) 0.81
(C) 0.90
(D) 1.06

51. If $y_p(x) = x\cos(2x)$ is a particular integral of the differential equation :

$y'' + \lambda y = -4\sin(2x)$, then $\lambda =$ ______.

$\lambda = -4 \frac{\sin 2x}{D^2 + \lambda}$

$-4 + 4$

(A) 4
(B) 2
(C) −2
(D) −4

52. The boundary-value problem: $y'' + y = 0$ with boundary conditions $y(0) = 1$, $y(2\pi) = 1$ has ______.

(A) no soluton.
(B) a unique solution.
(C) more than one, but finite number of solutions.
(D) an infinite number of solutions.

53. If $y(x)$ is the solution of the differential equation : $x^2\frac{d^2y}{dx^2} - 2y = 0$, $y(1) = 1$, $y(2) = 1$, then $y(3) =$ ______.

(A) $\frac{11}{7}$
(B) $\frac{17}{21}$
(C) $\frac{9}{7}$
(D) $\frac{11}{9}$

$D(D-1)$

$D^2 - D - 2 = 0$

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2)$

$(m-2)(m+1)$

$m = 2, -1$

$c_1 e^{2x} + c_2 e^{-x} = c_1 x^2 + \frac{c_2}{x}$

$c_1 e^2 + c_1 e^{-1} = 1$

$c_2 e^4 + c_1 e^{-2} = 1$

$\frac{9}{7} + \frac{2}{7}$

$\frac{11}{7}$

P.T.O.

54. What is the Laplace transform of the integral : $\int_0^t te^{-3t}\,dt$?

$(-t)^n e^{at} = \frac{d^n}{dt^n}$

$= -\frac{d}{dp}\frac{1}{p+3}$

$= -\frac{1}{(p+3)^2}$

60

(A) $\frac{1}{s(s+3)^3}$.

(B) $\frac{1}{s(s+3)^2}$.

(C) $\frac{1}{s^2(s+3)^2}$.

(D) $\frac{1}{s^3(s+3)}$.

55. If $P_n(x)$ denotes the Legendre polynomial of degree $n \geq 0$ and $1 + x^{10} = \sum_{k=0}^{10} \lambda_k P_k(x)$, then λ_5 = ________.

(A) 0.

(B) $\frac{3}{7}$

(C) $\frac{1}{2}$

(D) 1

56. The eigenvalues of the Strum-Liouville problem: $y'' + \lambda y = 0$, $y'(0) = 0$ and $y'(\pi) = 0$ are ________.

(A) $\frac{(2n-1)^2}{4}$ $(n \in \mathbb{N})$.

(B) $\frac{n^2}{4}$ $(n \in \mathbb{N})$.

(C) n^2 $(n \in \mathbb{N} \cup \{0\})$.

(D) $n^2\pi^2$ $(n \in \mathbb{N})$.

57. The solution of the partial differential equation : $xp + yq = z$ is ________ (ϕ is an arbitrary function).

$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$

(A) $\phi(x - y, x - z) = 0$

(B) $\phi\left(\frac{x}{y}, \frac{y}{z}\right) = 0$

(C) $\phi(x^2 - y^2, x^2 - z^2) = 0$

(D) $\phi(z, x^2 - y^2) = 0$

58. The coefficient of $\cos(5x)$ in the Fourier series expansion of the function $f(x) = |x|$ in the interval $(-\pi, \pi)$ is ________.

$a_0 + \Sigma (a_n \cos nx + b_n \sin nx)$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |x| \cos 5x \, dx$

$= \frac{2}{\pi}\left[|x| \frac{\sin 5x}{5} + \frac{1}{25}\cos 5x\right]_0^{\pi}$

$= \frac{2}{\pi}\left[-\frac{1}{25}\right] = -\frac{2}{25\pi}$

(A) $-\frac{4}{5\pi}$

(B) $\frac{4}{5\pi}$

(C) $-\frac{4}{25\pi}$

(D) $\frac{4}{25\pi}$

59. If $u(x, t)$ is the solution of the diffusion equation: $u_{xx} = u_t$ $(0 < x < \pi, t > 0)$ satisfying the conditions $u(0,t) = 0$ and $u(x,0) = 3\sin 2x$, then $u(x,t)$ = ________.

(A) $3e^{-t}\sin 2x$

(B) $3e^{-2t}\sin 2x$

(C) $3e^{-4t}\sin 2x$

(D) $3e^{-9t}\sin 2x$

60. Which one of the following options is true for the Cauchy problem : $2u_x + 3u_y = 5$ satisfying $u = 1$ on the line $y = \frac{3x}{2}$?

(A) It has a unique solution.

(B) It has more than one, but finite number of solutions.

(C) It has infinite number of solutions.

(D) It has no solution.

61. The optimal solution of the following linear programming (LP) problem :

Min $Z = 3x + 5y$ subject to $x + 3y \geq 3$, $x + y \geq 2$; $x, y \geq 0$ is ________. 10

(A) 5

(B) 7

(C) 10

(D) 11

$x=0, y=1$ $y=0, x=3$

$x=0, y=2$ $y=0, x=2$

62. If an artificial variable is present in the 'basic variable' column of the optimal simplex table, then the solution is said to be ________.

(A) Optimum

(B) Infeasible

(C) Unbounded

(D) Degenerate

63. Which one of the following statements is ***not*** necessarily ***true***?

(A) The dual of a dual LP problem is the original primal LP problem.

(B) If either the primal or the dual LP problem has an unbounded solution, then the solution to the other LP problem is infeasible.

(C) The objective function value of the dual LP problem at any feasible solution is always less than or equal to the objective function value of the primal LP problem at any feasible solution.

(D) If the primal LP problem has a feasble solution, but the dual LP problem does not have, then the primal LP problem will not have a finite optimum solution and vice versa.

64. Which one of the following methods is commonly used to solve assignment problems?

(A) Stepping Stone Method.

(B) North-West Corner Method.

(C) Vogel's Approximation Method.

(D) Hungarian Method.

65. The degeneracy in a transportation problem indicates that ________.

(A) dummy allocation(s) needs to be added.

(B) the problem has no feasible solution.

(C) the problem has multiple optimal solutions.

(D) (A) and (B) are true, but not (C).

66. In game theory, the outcome or consequence of a strategy is referred to as ________.

(A) penalty.

(B) reward.

(C) pay-off.

(D) end-game strategy.

67. For what range of value(s) of λ, the game with the following pay-off matrix is strictly determinable?

	Player B			
Player A	B_1	B_2	B_3	
A_1	λ	6	2	2
A_2	-1	λ	-7	-7
A_3	-2	4	λ	-2
	λ	6	2	

(A) $-2 \leq \lambda \leq 2$

(B) $-1 \leq \lambda \leq 2$

(C) $0 \leq \lambda \leq 2$

(D) $1 \leq \lambda \leq 2$

68. Which of the following statements is a valid logical inference?

(A) All mammals lay eggs. This is a mammal. Therefore, it lays eggs.

(B) All mammals lay eggs. This lays eggs. Therefore, it is a mammal.

(C) All birds lay eggs. This is a bird. Therefore, it lays eggs.

(D) All birds lay eggs. This lays eggs. Therefore, it is a bird.

69. For each $n \in \mathbb{N}$, the value of $\left(1+\frac{3}{1}\right)\cdot\left(1+\frac{5}{4}\right)\cdot\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2n+1}{n^2}\right)$ is ________.

$\frac{4}{1} \times \frac{9}{4} \times \frac{16}{9}$ $\qquad \frac{(n+1)^2}{(n)^2}$

(A) $\frac{(n+1)^2}{2}$

(B) $(n+1)^2$

(C) $\frac{(n+1)^3}{2}$

(D) $\frac{n(n+1)}{2}$

70. On the set $X = \{p, q, r, s, t\}$, consider the following relation

$R = \{(p,p), (p,q), (p,r), (p,s), (p,t), (q,q), (q,s), (q,t), (r,r), (r,t), (s,s), (s,t), (t,t)\}$.

Which one of the following statement is ***correct***?

(A) (X, R) is a Boolean algebra

(B) (X, R) is not a lattice.

(C) (X, R) is a complemented lattice.

(D) (X, R) is a distributed lattice.

71. Number of edges in the Hasse diagram representing the partial ordering $\{(n, m) : n \text{ divides } m\}$ on the set $X = \{1, 2, 3, 4, 6, 24, 36, 72\}$ is ________.

(A) 14

(B) 13

(C) 12

(D) 11

7+5+4+3+3+1+1

72. The minimization of the Boolean expression : $F(x, y) = y'(x' + y')(x + x'y)$ is ________ (The symbol "$'$" denotes the Boolean complement).

(A) xy

(B) xy'

(C) $x'y$

(D) $x'y'$

73. The generating function of the following recurrence relation :

$$a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 2 \text{ with } a_0 = a_1 = 1$$

is ________.

$a_2 = a_1 + a_0 = 2$

(A) $G_1(x) = \dfrac{x}{1-x-x^2}$.

(B) $G_2(x) = \dfrac{1+x}{1-x-x^2}$.

(C) $G_3(x) = \dfrac{1-x}{1-x-x^2}$.

(D) $G_4(x) = \dfrac{1}{1-x-x^2}$.

74. The number of elements in the adjacency matrix of a graph having 7 vertices is ________.

(A) 16

(B) 25

(C) 36

(D) 49

75. In a binary tree, the number of internal nodes of degree 1 is 5 and the number of internal nodes of degree 2 is 10. Then, the number of leaf nodes in the binary tree is ________.

(A) 10

(B) 11

(C) 12

(D) 15

5 + 15 = 20 = 2

76. Which one of the following statements is *false*?

(A) The complete graph K_n ($n \geq 3$) is a Hamiltonian graph.

(B) The complete graph K_n is a Eulerian graph, if and only if n is odd.

(C) The complete bipartite graphs $K_{m,n}$ are Hamiltonian, if and only if $m = n > 1$.

(D) The complete bipartite graphs $K_{m,n}$ are Eulerian, if and only if both m and n are odd.

77. On the poset $X = \{3, 5, 9, 15, 24, 45\}$ under the divisibility relation, which one of the following statements is *correct*?

(A) There exists a greatest element and a least element.

(B) There exists a greatest element but not a least element.

(C) There exists a least element but not a greatest element.

(D) There does not exist a greatest element and a least element.

78. Let G be a connected planar graph with 10 vertices. If the number of edges on each face is 3, then the number of edges in G is ________.

(A) 18 (B) 21

(C) 24 (D) 27

79. Maximization of an assignment problem is transformed into a minimization problem by ________.

(A) subtracting each entry in the table from the maximum value in that table.

(B) adding each entry in a column from the maximum value in that column.

(C) subtracting each entry in a column from the maximum value in that column.

(D) adding each entry in the table from the maximum value in that table.

80. Which one of the following statements is ***false*** in the case of a spanning tree of a graph G?

(A) It is a tree that spans the graph G.

(B) It is a subgraph of the graph G.

(C) It includes every vertex of the graph G

(D) It can be either cyclic or acyclic.

81. Which one of the following functions (defined for $z \neq 0$) can be extended countinuously to $z = 0$?

(A) $f_1(z) = \dfrac{\mathrm{Re}(z)}{z}$ (B) $f_2(z) = \dfrac{z}{|z|}$

(C) $f_3(z) = \dfrac{\mathrm{Re}(z^2)}{|z|^2}$ (D) $f_4(z) = \dfrac{z\,\mathrm{Re}(z)}{|z|}$

82. Which one of the following statements is ***true***? ($z = x + y,\ x, y \in \mathbb{R}$)

(A) $f_1(z) = x^2 + y^2 + 2ixy$ is analytic in $\mathbb{C}$. log (x+iy)

(B) $f_2(z) = \mathrm{Log}(z)$ is analytic in $\mathbb{C} \setminus \{0\}$. log z +

(C) $f_3(z) = \cos x \cosh y - i \sin x \sinh y$ is analytic in $\mathbb{C}$.

(D) $f_4(z) = \sin(7x + 5iy)$ is analytic in $\mathbb{C}$.

83. Which one of the following is ***true***? ($\mathrm{Arg}(z)$ denotes the principal value of the argument of z).

(A) $\mathrm{Arg}(z_1 z_2) = \mathrm{Arg}(z_1) + \mathrm{Arg}(z_2)$ for all $z_1, z_2 \in \mathbb{C} \setminus \{0\}$.

(B) $(z^c)^d = z^{cd}$, c, d complex and $z \in \mathbb{C} \setminus \{0\}$.

(C) $|\sin z| \leq 1$ for all $z \in \mathbb{C}$.

(D) There cannot exist a function analytic on an open set $\Omega \subset \mathbb{C}$ with real part $x^2 - 2y^2$.

84. Under the Möbius transformation $T(z) = iz$, the image of the straight line $y = x + 1$ in the xy-plane is ________.

(A) the circle $|w + 1| = 1$

(B) the straight line $u + v = -1$

(C) the straight line $u = \frac{1}{2}$

(D) the straight line $v = -2$

85. Let γ be the boundary of the region $\Omega = \{(x, y) : 0 < x < 1, 0 < y < 1 - x\}$ with counter-clockwise orientation. Consider the following integrals.

I. $\int_{\gamma} \text{Re}(z)dz = \frac{i}{2}$ $\qquad$ II. $\int_{\gamma} \text{Im}(z)dz = -\frac{1}{2}$

Choose the ***correct*** answer from the following options.

(A) Only I is correct.

(B) Only II is correct.

(C) Both I and II are correct.

(D) Both I and II are false.

$-\frac{1}{z}(1-e^z)^{-1}$

86. For the function $f(z) = \dfrac{1}{z(e^z - 1)}$ $(z \in \mathbb{C})$, the point $z = 0$ is ________. $-\frac{1}{z}(1-e^z$

(A) a removal singularity.

(B) a pole of order 2.

(C) an isolated essential singularity.

(D) a non-isolated essential singularity.

87. If $f : \mathbb{C} \to \mathbb{C}$ is a meromorphic function, analytic at $z = 0$ and satisfy $f\left(\frac{1}{n}\right) = \dfrac{n}{1+2n}$ for all $n \in \mathbb{N}$, then which one of the following is ***false***?

(A) $f(0) = \frac{1}{2}$

(B) $|f'(0)| = \frac{1}{4}$

(C) f has a simple pole at $z = -2$

(D) Residue of f at $z = -2$ is 2.

88. Let f be analytic in a neighborhood of the origin in the complex place . If

$$f^{(n)}(0) = (n-1)!(n+1)\left(\frac{n+1}{n}\right)^{n^2-1} \quad (n \in \mathbb{N}),$$

then the radius of the largest circle with centre at the origin inside which the Taylor series of f defines an analytic function is ________.

(A) $\frac{1}{e^2}$

(B) $\frac{1}{e}$

(C) e

(D) e^2

P.T.O.

89. If the following Laurent series expansion

$$\frac{1}{(z-1)(z-2)} = \sum_{-\infty}^{\infty} a_n z^n$$

is valid in the annulus $1 < |z| < 2$, then the coefficient of $\frac{1}{z^2}$ = ________. $\frac{7}{8}$

(A) -1 (B) 0

(C) $\frac{1}{2}$ (D) 1

90. $\int_{|z|=1} z^2 e^{1/z} \sin(1/z)dz$ = ________. $2\pi i\, b(0)$

(A) 0 (B) $\frac{2\pi i}{5}$

(C) $\frac{2\pi i}{3}$ (D) $\frac{\pi i}{2}$

91. Let $(X, \|\cdot\|)$ be a normed linear space and let d be the metric induced by the norm on X. If $x, y, z \in X$ and $\alpha \in \mathbb{R}$ or $\mathbb{C}$, then which one of the following is ***not*** necessarily ***true***?

(A) $d(x + z, y + z) = d(x,y)$ (B) $d(\alpha x, \alpha y) = |\alpha| d(x,y)$

(C) $d(d(x,y)x, d(x,y)y) = (d(x,y))^2$ (D) $d(\alpha x + y, \alpha y + x) = (\alpha - 1)d(x,y)$

92. On $\left(\ell^2, \|\cdot\|_2\right)$, define the linear functional f by

$$f(x) = \sum_{k=1}^{\infty} \frac{x_k}{3^k} \text{ for } x = (x_1, x_2, x_3, \ldots) \in \ell^2.$$

Then, which one of the following is ***correct***?

(A) $\|f\| = 1$ (B) $\|f\| = \frac{1}{2\sqrt{2}}$

(C) $\|f\| = \frac{\pi}{\sqrt{6}}$ (D) $\|f\| = \infty$

93. On the linear space $C([0, 1])$, define the norms :

$$\|f\|_\infty = \sup_{x \in [0,1]} |f(x)|, \; \|f\|_1 = \int_0^1 |f(t)|\, dt, \; \|f\|_2 = \left(\int_0^1 |f(t)|^2\, dt\right)^{\frac{1}{2}} \quad (f \in C([0,1]))$$

and consider the following statements.

I. $(C([0, 1]), \|\cdot\|_\infty)$ is a Banach space.

II. $(C([0, 1]), \|\cdot\|_1)$ is a Banach space.

III. $(C([0, 1]), \|\cdot\|_2)$ is a Banach space.

Choose the ***correct*** answer from the following options.

(A) Only I is true.

(B) Only III is true.

(C) Only I and III are true.

(D) Only II and III are true.

94. On a finite dimensional normed linear space X, consider the following statements.

I. All norms on X induce the same topology on X.

II. Weak convergence of a sequence in X imply the convergence of the sequence in any norm of X.

III. Any linear operator on X is not necessarily bounded on X.

Choose the ***correct*** answer from the options given below.

(A) Only III is true.

(B) Only I and III are true.

(C) Only II and III are true.

(D) Only I and II are true.

95. Which one of the following statements is ***false***?

(A) The dual space of $\left(\ell^1, \|\cdot\|_1\right)$ is isometrically isometric to $\left(\ell^\infty, \|\cdot\|_\infty\right)$.

(B) The space $\left(\ell^1, \|\cdot\|_1\right)$ is reflexive.

(C) The space $\left(\ell^1, \|\cdot\|_1\right)$ is separable.

(D) The dual space of $\left(c_0, \|\cdot\|_\infty\right)$ is isometrically isometric to $\left(\ell^1, \|\cdot\|_1\right)$.

96. If X, Y are normed linear spaces and $T : X \rightarrow Y$ is a surjective, bounded linear operator, then which one of the following statements is ***true***?

(A) T is always an open map.

(B) T is an open map, if X is a Banach space.

(C) T is an open map, if Y is a Banach space.

(D) T is an open map, if both X and Y are Banach spaces.

97. Let X, Y be normed linear spaces and let F be a collection of bounded linear operators from X to Y. Consider the following statements.

I. If F is uniformly bounded, then F is point-wise bounded.

II. If F is point-wise bounded and Y is a Banach space, then F is uniformly bounded.

III. If F is point-wise bounded and X is a Banach space, then F is uniformly bounded.

Choose the ***correct*** statement(s) from the options given below.

(A) Only III is true.

(B) Only I and II are true.

(C) Only I and III are true.

(D) I, II and II are true.

98. With the usual inner product on $\mathbb{R}^3$, let the set $S = \{u, v, w\}$ be an orthonormal basis for $\mathbb{R}^3$. If $u = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ and $v = (0, 0, 1)$, then $w =$ ________. $\langle u, v \rangle$ ✓

(A) (0, 1, 0)

(B) (1, 0, 0)

(C) $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

(D) ✓ $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

99. Let $\mathcal{H}$ be a Hilbert space over $\mathbb{R}$. If $x, y \in \mathcal{H}$ with $\|x\| = 4$, $\|y\| = 3$ and $\|x - y\| = 3$, then $\langle x, y \rangle =$ ________ ($\langle \cdot, \cdot \rangle$ denotes the inner product induced by the norm on $\mathcal{H}$).

$\sqrt{\cancel{2}4 \cdot 16 + 9 - 3}$

$= 25$

(A) 4

(B) 6

(C) 8

(D) 10

100. Let ϕ be a bounded linear functional on $\left(\ell^2, \|\cdot\|_2\right)$ defined by $\phi(x) = x_1 + x_3$ for all $x = (x_1, x_2, x_3, \ldots) \in \ell^2$. Then, the unique element $y \in \ell^2$ representing ϕ as given by the Riesz representation theorem for Hilbert spaces is ________.

(A) (0, 1, 0, 1,...)

(B) (1, 0, 1, 0,...)

(C) (0, 0, 1, 1,...)

(D) (1, 1, 0, 0,...)